Traffic State Estimation from Aggregated Measurements with Signal Reconstruction Techniques

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The estimation of the state of traffic provides a detailed picture of the conditions of a traffic network based on limited traffic measurements and, as such, plays a key role in intelligent transportation systems. Most of the existing state estimation algorithms are based on Kalman filtering and its variants, which, starting from the current estimate, predict the future state and then correct it on the basis of new measurements. Most often, traffic measurements are aggregated over multiple time steps, and this procedure raises the question of how to best use this information for state estimation. A standard approach that performs the correction only at the time step when the aggregated measurement is received is suboptimal. Reconstructing the high-resolution measurements from the aggregated ones and using them to correct the state estimates at every time step are proposed. Several reconstruction techniques from signal processing, including kernel regression and a reconstruction approach based on convex optimization, were considered. The proposed approach was evaluated on real-world NGSIM data collected at Interstate 101, located in Los Angeles, California. Experimental results show that signal reconstruction leads to more accurate traffic state estimation as compared with the standard approach for dealing with aggregated measurements.

It has been estimated that traffic congestion costs the world economy hundreds of billions of dollars each year, increases pollution, and has a negative impact on the overall quality of life in metropolitan areas. In order to solve this emerging problem, transportation departments increasingly rely on systems for real-time traffic control and management known as intelligent transportation systems (ITS). In addition to real-time operations management, ITS are valuable as a source of data for transportation planning and traveler information systems. The success of the ITS greatly depends on the ability to measure traffic conditions and estimate traffic states at a fine spatial and temporal scale.

Because of the importance of traffic state estimation, the transportation research community has developed and evaluated many algorithms for estimation of traffic variables such as traffic flow, speed, and density. The spatiotemporal evolution of these traffic quantities is described by using deterministic traffic models and is estimated from measurements with the Kalman filter (KF) algorithm and its extensions (*1*). For instance, one of the most commonly used traffic models for traffic state estimation is the cell transmission model (CTM) (*2, 3*), the first-order traffic model that can be used for estimation of traffic density (*4*) or traffic speed (*5*). Since traffic models are typically highly nonlinear, the basic KF, designed for linear problems, cannot be used. This problem prompted development of methods for nonlinear state estimation that are extensions of the basic Kalman filtering idea. For instance, the extended KF (*6*) was successfully implemented in RENAISSANCE, a real-time freeway traffic network surveillance tool (*7*), as well as in the second-order traffic flow model METANET (*8*). When the traffic model is highly nonlinear, alternative approaches such as unscented KFs (*9*), mixture KFs (*4*), particle filters (*10*), and ensemble KFs (*5*) have been shown to perform quite well.

All KF-based approaches for traffic state estimation consist of prediction and correction steps. In the prediction step, starting from the current state estimate, the traffic state in the next time step is predicted by the traffic model. In the correction step, if new measurements are available, the predicted traffic state is corrected according to the measurements. However, the problem is that the traffic measurements provided by many types of sensors, including the ubiquitous loop detectors, are most often aggregated across multiple time steps, making it difficult to know how to use them for state estimation. A standard way to address this issue is to use the aggregated measurements for correction only at the time steps when they become available and to not correct the predicted states in the remaining time steps. However, this approach is suboptimal since it could lead to large uncertainty and low accuracy of state estimation.

To solve this problem, Schreiter et al. recently proposed using the aggregate measurements for correction at all time steps during the aggregation period by postulating that measurements at all time steps are equal to the corresponding aggregated measurements (*11*). Although this approach can be very successful when traffic is in the free-flow regime, it is not the best choice during the congested regime or during transitions between the regimes. The use of signal reconstruction techniques is explored to re-create the measurement at every time step from the available aggregated measurements. In this light, the approach proposed by Schreiter et al. can be interpreted as stepwise reconstruction of original measurements with the aggregated measurements (*11*). In addition to the stepwise reconstruction, a few more signal reconstruction algorithms with increased complexity, ranging from linear interpolation to spline approximation to treating signal reconstruction as a convex optimization problem, are explored. Finally, all these approaches are evaluated on the high-quality NGSIM traffic data.

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PROBLEM DESCRIPTION

The following nonlinear dynamical system is considered:

$$\mathbf{x}_t = M\left(\mathbf{x}_{t-1}\right) + \eta_t$$
$$\mathbf{z}_t = H\left(\mathbf{x}_t\right) + \xi_t \tag{1}$$

where

$\mathbf{x}_t$ = vector of state variables,
$\mathbf{z}_t$ = vector of measurements at time step t,
mapping M = state transition model,
H = measurement model,
η_t = state process Gaussian noise with zero mean and covariance Q_t, and
ξ_t = observation Gaussian noise with zero mean and covariance R_t.

In the traffic domain, the vector **x** represents a vector of traffic variables such as speed, density, or flow, and vector **z** represents sensor readings, in most cases of speed or flow. According to the CTM approach (*2, 3, 5*), which is used here, roads in the traffic network are divided into spatial cells of arbitrary but preferably equal or similar lengths. In the case of Eulerian traffic sensors, such as fixed loop detectors, cells are typically positioned in such a way that the sensors are located at their downstream end and only a small subset of cells contains a sensor. The time is discrete, divided into steps whose length is constrained by the Courant–Friedrichs–Lewy conditions, stating that a moving vehicle cannot traverse more than one spatial cell during one time step (*12*). For example, if the free-flow speed equals 60 km/h and a stretch of freeway is discretized into spatial cells each 100 m long, the time step must not be longer than 6 s.

The traffic state estimation problem is to estimate a sequence of true states **x,** which are not directly observable, given a sequence of measurements **z.** If we choose functions $M(\cdot)$ and $H(\cdot)$ to be linear, the well-known KF closed-form solution can be used for this problem (*1*); otherwise, there are many approaches that have been proposed to deal with the nonlinearity of transition or observation functions, or both (*5, 6, 9*). All KF methods work in a similar way, by iteratively performing prediction and correction steps. In the prediction step, the next state $\mathbf{x}_t$ is predicted given knowledge about the current state $\mathbf{x}_{t-1}$, and in the correction step, measurements $\mathbf{z}_t$ of the current system state are used to reestimate (i.e., correct) the prediction. For various reasons, observations might not be available at every time step, in which case only the current state can be predicted without the correction step.

The dynamical system (Equation 1) implies that the measurements $\mathbf{z}_t$ are obtained during time step t. However, traffic measurements are usually aggregated over a period of time to account for the inherent signal noise or to allow easier transmission and storage of large amounts of measured data. The aggregated measurements are reported every Δ time steps, where Δ is referred to as the aggregation period. As a result, instead of the observations $\mathbf{z}_t$, only the aggregated measurements $\mathbf{y}_T$ at time steps that are multiples of Δ, $T \in \{\Delta, 2\Delta, \ldots\}$ are available. Vector $\mathbf{y}_T$ available at time T is an aggregation of $\mathbf{z}_t$-values during the previous Δ time steps, $T - \Delta < t \le T$. For instance, in case of the volume measurement, the aggregated volume is the sum of all volume measurements during the aggregation period, and in the case of the speed measurement, the aggregated speed can be the average speed during the aggregation period. To consider the second case in more detail, the aggregated speed can be calculated as follows:

$$\mathbf{y}_T = \frac{1}{\Delta} \sum_{t=T-\Delta+1}^{T} \mathbf{z}_t \tag{2}$$

Combining Equations 1 and 2 results in

$$\mathbf{y}_T = \frac{1}{\Delta} \sum_{t=T-\Delta+1}^{T} H\left(\mathbf{x}_t\right) + \frac{1}{\Delta} \sum_{t=T-\Delta+1}^{T} \xi_t \tag{3}$$

where, by assuming that the observation noise ξ_t is sampled independent and identically distributed, it follows that the noise variance drops by a factor of $1/\Delta$ as compared with the original measurements. During free flow, this result has a positive effect since $H(\mathbf{x}_t)$ is stable and measurement noise is reduced. However, during the transition or congestion periods, the gain in measurement noise is offset by a loss of information about the fine-scale changes in the traffic state.

Figure 1 illustrates the measurement aggregation. Instead of the true measurement signal during period Δ, only its mean value y_T at the end of that period is given.

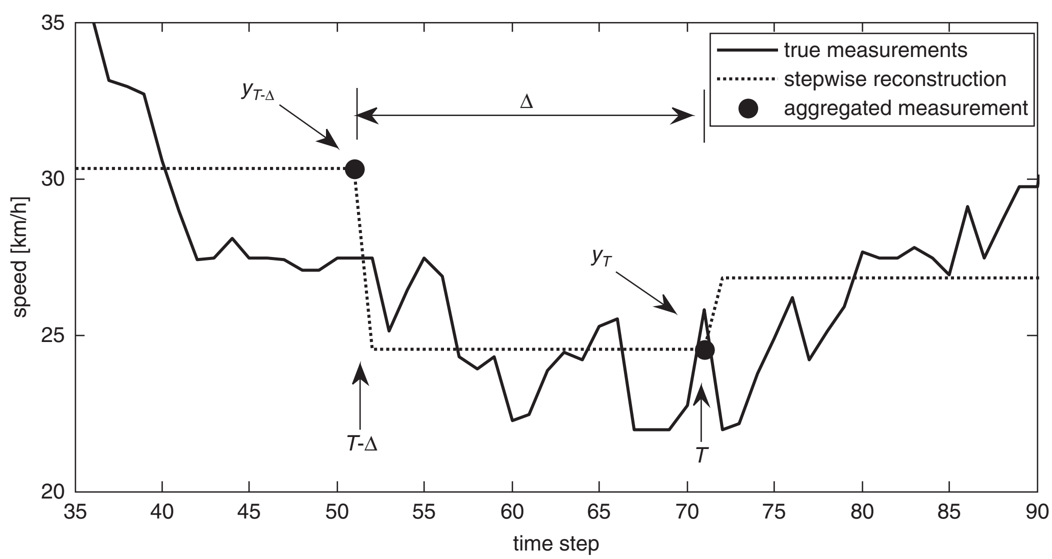

FIGURE 1 Problem description.

The main question when one is dealing with the aggregated measurements is how to use them for state estimation. The problem with missing and aggregated measurements has also been recognized in other fields, such as hydrology and meteorology (*13*), where KF equations are modified to incorporate aggregated measurements. In economics, the same problem occurs when aggregated univariate (*14*) and multivariate (*15*) economic time series are analyzed, and it is addressed by augmenting the state space. However, state space augmentation would be too cumbersome and too computationally costly for traffic state estimation, considering the fact that spatiotemporal traffic models are nonlinear and highly dimensional. Therefore, in this work a much simpler, but still effective, signal reconstruction approach for dealing with aggregated traffic measurements is examined.

The reconstructed signal at time step t is denoted $\hat{\mathbf{z}}_t$, and the reconstruction function $A(\cdot)$, such that

$$\hat{\mathbf{z}}_t \equiv A(\mathbf{y},\ t) \tag{4}$$

where $\mathbf{y}$ is the set of aggregated measurements until time T, $\mathbf{y} = \{\mathbf{y}_1, \ldots, \mathbf{y}_T\}$, such that $t \le T$. The reconstructed measurements $\hat{\mathbf{z}}_t$ are used in the KF correction step instead of the unknown true measurements $\mathbf{z}_t$ defined in Equation 1. Depending on the form of the function $A(\cdot)$, different reconstructions of the aggregated measurement can be obtained, and in the following two sections several different choices for the reconstruction function are described.

EXISTING METHODS

A typical way of dealing with the aggregated measurements is to apply the correction step only at the time steps in which the aggregate measurements become available and to use them directly as measurements, $\hat{\mathbf{z}}_t = \mathbf{y}_T$, $t = T$. Consequently, the aggregated measurement is used to correct the system state at the end of the corresponding aggregation period, whereas during the remaining $\Delta - 1$ time steps only the prediction steps are used. This method is referred to as the classic approach, since it is the most commonly used approach in existing traffic state estimation algorithms. For this approach, the reconstruction function is defined as follows:

$$A(\mathbf{y},\ t) = \begin{cases} \mathbf{y}_t & \text{if } t = T \\ NaN & \text{otherwise} \end{cases} \tag{5}$$

There are several issues with this method. First, $\mathbf{z}_t$ at time steps $t = \Delta, 2\Delta, \ldots, K\Delta$ does not necessarily equal $\mathbf{y}_T$. Second, information about $\mathbf{y}_T$ is used at only one of the Δ steps, although $\mathbf{y}_T$ contains information about the aggregated values at all Δ steps. As a result, the state estimation will be suboptimal, characterized by sudden jumps in the estimates and by increasing uncertainty at time steps between the correction steps.

To address this problem, Schreiter et al. (*11*) recently proposed a method to reconstruct all $\mathbf{z}_t$ vectors during the aggregation period. Specifically, instead of using the aggregated measurements $\mathbf{y}_T$ to correct only the predicted state at time step T, as in the classic approach, the method corrects all previously predicted states. The reconstruction function by Schreiter et al. (*11*) is defined as follows:

$$A(\mathbf{y},\ t) = \mathbf{y}_T \qquad T - \Delta < t \le T \tag{6}$$

and represents a stepwise reconstruction of the aggregated signal (see Figure 1). The authors showed experimentally that this approach outperforms the classic approach from Equation 5. The stepwise reconstruction from Equation 6 is characterized by potentially large discontinuities after every aggregation period. This feature could lead to suboptimal traffic state estimation, especially during the transition periods when the traffic undergoes regime changes or within congested periods. This method, unlike the classic approach, exhibits estimation delay of one aggregation period, since it waits until the new aggregated measurement $\mathbf{y}_T$ is acquired at time step T to reconstruct the measurements $\mathbf{z}_t$ and estimate the traffic states $\mathbf{x}_t$ at time steps $T - \Delta < t \le T$. This method is referred to as the stepwise approach.

The idea to reconstruct the original measurements and use them in the correction step of a KF can be further improved by employing more sophisticated signal reconstruction techniques. Several approaches for signal reconstruction and how they affect the accuracy of traffic state estimation from aggregated measurements are studied here.

SIGNAL RECONSTRUCTION FROM AGGREGATED MEASUREMENTS

The stepwise approach from Equation 6 is the simplest measurement reconstruction scheme from the aggregated data. Now several more sophisticated signal reconstruction approaches will be considered, including three standard piecewise interpolation techniques (linear, cubic spline, and cubic Hermite spline interpolation), as well as kernel regression and a convex optimization approach. During the interpolation process for all approaches other than the convex optimization, the aggregated measurement $\mathbf{y}_T$ will be assumed to be in the middle of the aggregation period (Figure 2*a*). More formally, when at time step T, it is assumed that $\mathbf{y}_T$ was obtained at time step $T - \Delta/2$. With this assumption, $\mathbf{y}_T$ can be treated like a sample from the measurement time series $\{\mathbf{z}_t\}$ filtered with a centered window of length Δ. The following signal reconstruction techniques were studied.

Piecewise Linear Interpolation

Linear approximation by a straight line between the aggregated measurements (Figure 2) is

$$A(\mathbf{y},\ t) = \mathbf{y}_{T-\Delta} + (\mathbf{y}_T - \mathbf{y}_{T-\Delta})\frac{\left(t - T + \frac{3\Delta}{2}\right)}{\Delta} \qquad \text{for } t \in \left(T - \frac{3\Delta}{2}, T\right. \tag{7}$$

It should be observed that when $\mathbf{y}_{T+\Delta}$ becomes available, the signal is reconstructed within the interval $t \in (T - \Delta/2, T + \Delta]$. As a result, the old signal reconstruction in the interval $t \in (T - \Delta/2, T]$ will be modified (compare this interval in Figure 2, *a* and *b*), which typically results in a more accurate reconstruction. This observation will be important when the delay allowed in the traffic state estimation is discussed in the following section.

Piecewise Cubic Spline Interpolation

As an alternative to interpolation by a straight line, cubic spline interpolation reconstructs the signal as piecewise cubic splines. Spline interpolation uses low-degree polynomials (of degree 3 in these

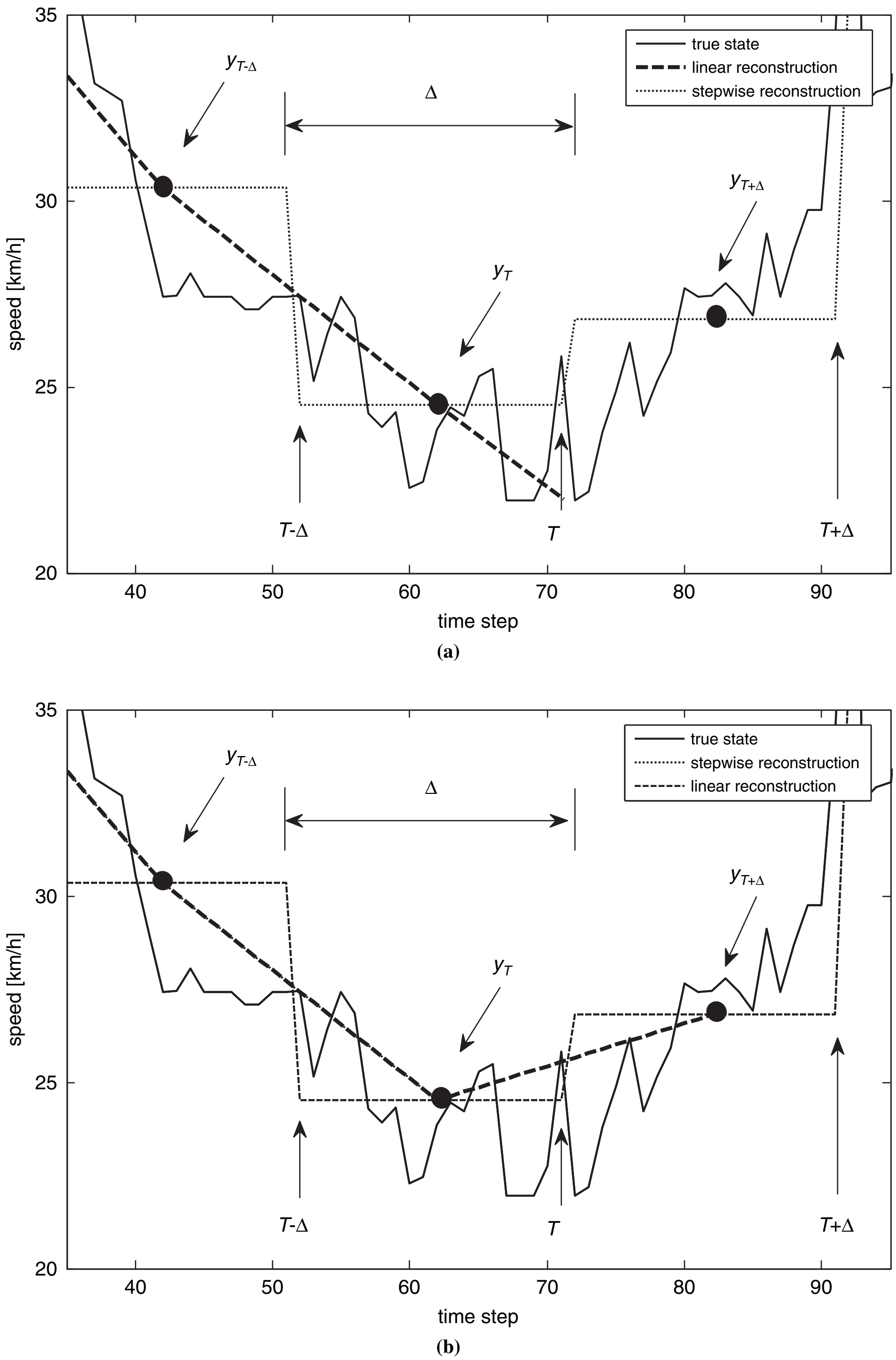

FIGURE 2 Piecewise linear interpolation approach: (*a*) after receiving measurement at time T and (*b*) after receiving measurement at time $T + \Delta$.

experiments) in each of the aggregation intervals and chooses them such that they fit smoothly together. Given T aggregated measurements, the spline function fits these points with the spline curve such that it is made out of $T-1$ cubic polynomials of the following form:

$$f_i(t) = a_i + b_i t + c_i t^2 + d_i t^3 \qquad i = 1, 2, \ldots, T-1 \tag{8}$$

where a_i, b_i, c_i, and d_i are coefficients of the ith polynomial found during training. The reconstruction function is given by

$$A(\mathbf{y}, t) = f_i(t) \qquad \text{if } t \in \left(i \cdot \Delta - \frac{\Delta}{2}, i \cdot \Delta + \frac{\Delta}{2} \right) \tag{9}$$

where i is defined as in Equation 8. The stepwise method is a special case of this approach, obtained by piecewise 0-degree polynomials. Also, upon receiving new aggregated measurement $\mathbf{y}_{T+\Delta}$, the whole signal from time $t = 1$ until time $t = T + \Delta$ needs to be reconstructed.

Cubic Hermite Spline Interpolation

Cubic Hermite spline interpolation is similar to the previously introduced cubic spline interpolation, the difference being that the polynomials (Equation 8) are in the Hermite form (*16*).

Kernel Regression Approach

Kernel regression is a common smoothing technique (*17*) that reconstructs a signal as a weighted average of the neighboring aggregated measurements:

$$A(\mathbf{y}, t) = \frac{\sum_i \mathbf{y}_i k(i, t)}{\sum_i k(i, t)} \qquad i \in \{\Delta, 2\Delta, \ldots, T\Delta\} \tag{10}$$

where $k(\cdot,\cdot)$ is a kernel function. The common choice for the kernel function is the Gaussian kernel (used here), defined as follows:

$$k(i, t) = \exp\left(-\frac{(i-t)^2}{\sigma^2} \right) \tag{11}$$

where σ is a kernel width parameter. For smaller values of σ only the closest neighbors are considered, whereas for larger σ the reconstructed signal becomes very smooth.

Convex Optimization Approach

Finally, the interpolation problem is defined as a convex optimization problem. Convex optimization relates to a class of nonlinear optimization problems in which both the objective to be minimized and the constraints are convex. Convex optimization problems are attractive because a large class of these problems can now be efficiently solved (*18*). Given the set of aggregated measurements $\mathbf{y}$, it is proposed to find smooth estimates $\hat{\mathbf{z}}$ by solving the following convex problem:

$$\min_{\hat{\mathbf{z}}} \sum_{t=1}^{T\Delta-1} \left(\hat{\mathbf{z}}_t - \hat{\mathbf{z}}_{t+1} \right)^2$$

subject to

$$\frac{1}{\Delta} \sum_{t=(j-1)\cdot\Delta+1}^{j\cdot\Delta} \hat{\mathbf{z}}_t = \mathrm{y}_j \qquad j = 1, \ldots, T \tag{12}$$

The objective function ensures that the approximation is as smooth as possible, and the constraint ensures that the mean value of estimated measurements within every aggregation interval is equal to the corresponding aggregated measurement. In this case, there is no analytical form for the reconstruction function $A(\cdot)$; rather, the solution of the optimization problem is used instead:

$$A(\mathbf{y}, t) = \hat{\mathbf{z}}_t \tag{13}$$

where, with a slight abuse of notation, $\hat{\mathbf{z}}_t$ is the tth element of the solution of the convex problem (Equation 12).

ALGORITHM

To discuss the technical details of the traffic state estimation algorithm based on Kalman filtering, which employs the proposed signal reconstruction techniques, it will be assumed that the newest aggregated measurement was received at time T and that the measurement vectors $\mathbf{z}_t$ have been reconstructed up to time point T. It will also be assumed that all the estimated states $\mathbf{x}_t$ up to time point T are saved. Once the aggregate at time $T + \Delta$ becomes available, the reconstruction of the entire measurement sequence is repeated up to time point $T + \Delta$. In the case of the stepwise reconstruction, the reconstructed $\hat{\mathbf{z}}_t$ at time steps $t < T$ will remain unchanged. Therefore, Kalman filtering is performed from time step T to $T + \Delta$ by using the reconstructed signal $\hat{\mathbf{z}}_t$. With all other reconstruction techniques proposed in the previous section, the reconstructed $\hat{\mathbf{z}}_t$ at time steps $t < T$ will be modified.

Since the states $\mathbf{x}_t$ during time period $[T - \Delta, T]$ were estimated by using the old reconstructed signal, one can back up in time and reestimate the states for period $[T - \Delta, T]$ by using the new reconstructed measurement. This reestimation of the states is done to better prepare the KF for the current estimation period $[T, T + \Delta]$. Once the state reestimation for period $[T - \Delta, T]$ is completed, the state estimation proceeds by using the reconstructed measurements for period $[T, T + \Delta]$.

The algorithm for the KF with the proposed signal reconstruction techniques is summarized in the following (this algorithm requires an estimation delay of one aggregation interval, Δ). Time step T is assumed and further that the state of the KF at time step $T - \Delta$ is stored in memory.

Repeat:

1. Wait until aggregated measurement $\mathbf{y}_{T+\Delta}$ becomes available.
2. Reconstruct the measurement up to time step $T + \Delta$ using all available aggregated measurements.
3. Load from memory the KF state at time step $T - \Delta$.
4. Reestimate the states by using the KF from time step $T - \Delta + 1$ to T.
5. Store to memory the KF state at time step T.
6. Estimate system states from time steps $T + 1$ to $T + \Delta$ by using the KF (and report this to the user).
7. Set $T \leftarrow T + \Delta$.

Depending on the allowed delay in making the traffic state estimation, several versions of the algorithm can be obtained. In addition to the foregoing one-interval delay algorithm, it is interesting to consider the offline version of the algorithm, which allows an arbitrary delay in the state estimation. This mode of operation is called the *analysis mode.* In this mode, it is assumed that a sequence of historical aggregated measurements is given and that the goal is to estimate the sequence of historical traffic states. This mode is useful when the historical data are analyzed and is also used as a benchmark for the one-interval delay mode, since better results with the use of the analysis mode are expected. Algorithmically, the difference between this mode and the foregoing algorithm is that only Steps 2, 6, and 7 are iterated.

VELOCITY CELL TRANSMISSION MODEL

In order to test the proposed signal reconstruction approaches for dealing with aggregated measurements, the velocity cell transmission (CTM-v) model (*5*) was employed because of its simplicity. It is based on the Lighthill–Whitham–Richards model (*19, 20*) and is defined as follows:

$$x_t^i = x_{t-1}^i - \frac{\Delta t}{\Delta x}\left(g\left(x_{t-1}^i, x_{t-1}^{i+1}\right) - g\left(x_{t-1}^{i-1}, x_{t-1}^i\right)\right) \qquad i = 1, 2, \ldots, L \quad (14)$$

where

x_t^i = speed at ith cell at time step t,
L = total number of cells, and
g = numerical flow function defined as follows:

$$g\left(x_1, x_2\right) = \begin{cases} R\left(x_2\right) & \text{if } x_1 \le x_2 \le x_c \\ R\left(x_c\right) & \text{if } x_1 \le x_c \le x_2 \\ R\left(x_1\right) & \text{if } x_c \le x_1 \le x_2 \\ \max\left(R\left(x_2\right), R\left(x_1\right)\right) & \text{if } x_1 \ge x_2 \end{cases} \quad (15)$$

where $x_c = x_{max}/2$, $R(x) = x^2 - x_{max}x$, and x_{max} is the maximal speed allowed by the CTM-v model. Boundary conditions before the first cell and after the last cell modeled by Equation 14 are modeled as random walk (*7*), such that two ghost cells are added at the beginning and at the end of the road segment:

$$x_t^0 = x_{t-1}^0 + \xi_t^0$$
$$x_t^{M+1} = x_{t-1}^{L+1} + \xi_t^{L+1} \quad (16)$$

The CVM-v model represents the model transition function $M(\cdot)$ from Equation 1 by combining Equations 15 and 16.

In the CTM-v model, it is assumed that sensors are measuring speed (e.g., they are double loop detectors) at a subset of cells. As a result, the observation matrix H from Equation 1 is a linear function of system variables, since the state of the system is directly measured. The measurement noise variance R_t is assumed to be constant in time and is denoted simply R.

ENSEMBLE KALMAN FILTER

The ensemble KF (EnKF) was first introduced by Evensen as an alternative to the extended KF, which performs poorly when the state transition function is highly nonlinear (*21*). The EnKF belongs to a group of suboptimal estimators that use Monte Carlo or ensemble integration. The EnKF uses a collection of state vectors (called the ensemble members of system states) to propagate the state forward in time and to compute the mean and covariance needed for the correction step. The covariance estimated in this way is used to compute the Kalman gain, and the correction step equation stays the same as in the traditional KF. The EnKF algorithm employing the CTM-v model is summarized in the following steps. In Step 1, samples that are generated represent the prior knowledge about the initial state and this step represents initialization of the system. Steps 2 through 4 represent the prediction phase, whereas Steps 5 and 6 represent the correction phase.

1. Generate N ensemble members of system states s_0^n, by drawing N samples from a Gaussian distribution, where $n = 1, 2, \ldots, N$, and index 0 denotes the initial time step.
2. Make a prediction using the CTM-v model:

$$\hat{s}_t^n = M\left(s_{t-1}^n\right) + \eta_t^n$$

3. Compute the mean of the ensemble:

$$x_t = \frac{1}{N}\sum_{n=1}^{N}\hat{s}_t^n$$

4. Use the mean of the ensemble to compute the covariance of the predicted state:

$$P_t = \frac{1}{N-1}E_t\left(E_t\right)^T$$

where matrix E_t is defined as $E_t = [\hat{s}_t^1 - x_t, \ldots, \hat{s}_t^N - x_t]$.

5. Calculate the Kalman gain as follows:

$$K_t = P_t\left(H\right)^T\left[H\ P_t\left(H\right)^T + R\right]^{-1}$$

6. Use the measurement to obtain a new ensemble:

$$s_t^n = \hat{s}_t^n + K_t\left[z_t - H\hat{s}_t^n + \xi_t^n\right]$$

7. Go to Step 2.

DATA SET

The EnKF with the described signal reconstruction approaches was tested on the NGSIM data set (*22*) collected at Interstate 101 (Hollywood Freeway), located in Los Angeles, California. This stretch of highway is 640 m long and consists of five lanes and one on- and one off-ramp. Trajectories of all vehicles were collected on June 15, 2005, between 7:50 a.m. and 8:05 a.m. To collect such high-quality data, eight video cameras were used to monitor this section of highway, and from the recorded video data, coordinates for each vehicle were extracted every 0.1 s.

Compared with the typical double loop detector data, where length of highway is usually several kilometers and distance between detectors is hundreds of meters, the portion of the highway in the NGSIM data set is very short in both time and space. To make this data set more suitable for traffic state estimation, the leftmost lane was divided into 32 cells 20 m long with virtual detectors placed every 100 m. As a result, starting with the Cell Number 2, every fifth cell contains a virtual sensor that provides speed measurements. The whole 15-min time interval was divided into time steps of 0.6 s, which is consistent with the Courant–Friedrichs–Lewy conditions (*12*). The first 100 time steps were discarded because of missing data. As a result, the final discretization of time and space amounts to 32 spatial cells and 1,400 time steps. To simulate the aggregation, it was assumed that sensors report aggregated speed every Δ time steps. The reported $\mathbf{y}_T$ aggregated values were obtained as the average speed during the Δ time steps plus a random Gaussian noise with mean zero and variance 1.

EXPERIMENTAL SETUP

The study objective was to estimate the true traffic state available in the original NGSIM data given the aggregated measurements and to preprocess them by using various signal reconstruction approaches detailed earlier. The reported performance measure is mean absolute error (MAE), defined as follows:

$$\text{MAE} = \frac{1}{L \cdot K} \sum_{i,t} \left| x_{i,t}^{\text{estim}} - x_{i,t}^{\text{true}} \right| \qquad (17)$$

where $x_{i,t}^{\text{estim}}$ and $x_{i,t}^{\text{true}}$ are estimated and true speed, respectively, for the ith spatial cell at time step t, where $t = 1, 2, \ldots, K$, and $i = 1, 2, \ldots, L$. Variables K and L represent the total number of time steps and spatial cells, respectively. The embedded MATLAB implementations were used for linear, cubic spline, and cubic Hermite spline interpolations (*interp1* function), kernel regression was implemented in MATLAB, and the convex optimization approach was solved by using CVX, the MATLAB toolbox for disciplined convex programming (*23*). The experiments were repeated five times, and the average MAE and the standard deviation were reported.

The initialization of EnKF was done such that the ensemble members are sampled from a Gaussian distribution with mean 60 km/h and variance 10 km/h. The mean and the variance were chosen on the basis of the documentation provided with the data set. The maximum speed on this stretch of highway was determined to be 105 km/h and the ensemble size was set to 200. The remaining parameters of the EnKF model were set to the following values: measurement noise variance, 1 km/h; state noise variance, 100 km/h for ghost and 5 km/h for all other cells. These parameter values were obtained by model calibration.

RESULTS

First, the performance of two existing signal reconstruction methods (classic and stepwise) was compared with the performance of the five proposed reconstruction techniques. Separate experiments were run for different lengths of the aggregation interval Δ, where $\Delta \in \{1, 2, 4, 10, 20, 40\}$. When $\Delta = 1$, the aggregation is not performed and the original, virtual loop detector measurements are used in the correction step. Although this is not a realistic setting, it is very useful as a benchmark. Results for the one-interval delay and the analysis mode are presented in Table 1.

Confirming the results from Schreiter et al. (*11*), the stepwise approach consistently achieves better performance than the classic approach, demonstrating the benefits of the measurement reconstruction. However, although the stepwise and classic approaches achieve reasonable results for shorter aggregation intervals in both modes of operation, for longer intervals the proposed signal reconstruction methods work better. For example, when the aggregation interval Δ is set to 40, the reconstruction approach based on convex optimization achieves 10% improvement over the stepwise reconstruction. When the analysis and the one-interval modes are compared, the reported MAE is nearly the same for all interpolation methods, except for cubic spline interpolation. The reason is poor interpolation of the cubic spline method for interval $t \in (T - \Delta/2, T]$, stemming from the use of higher-degree polynomials. However, in the analysis mode only interpolation is performed, which leads to improved performance. The performance of the convex optimization approach and the linear interpolation approach is very similar in all settings.

For the remaining experiments, the aggregation interval was fixed to $\Delta = 20$. To get better insight into the state estimation performance, Figure 3 shows the reconstructed measurement signal for the cell with a sensor obtained with different reconstruction methods. The difference between the proposed approaches and the stepwise approach is clearly visible; proposed approaches follow the true (unobserved) measurement much closer and without sudden jumps.

For further insight, Figure 4 demonstrates the difference in speed estimation between the stepwise and cubic Hermite interpolation approaches on two representative types of cells, one with and one without a virtual sensor. When the approaches are applied to the cell with a sensor, the step function in the stepwise approach does not model traffic speed well during the transition periods, whereas the cubic approach handles these situations well. Both approaches fail

TABLE 1 MAEs for Different Aggregation Intervals

Mode	Δ	Classic	Stepwise	Linear Interpolation	Spline Interpolation	Hermite Interpolation	Optimization	Kernel Regression
One-interval delay	1	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07
	2	4.90 ± 0.05	4.74 ± 0.01	4.74 ± 0.06	5.02 ± 0.08	4.82 ± 0.05	4.75 ± 0.07	4.92 ± 0.03
	4	5.30 ± 0.07	4.79 ± 0.06	4.80 ± 0.05	5.05 ± 0.04	4.83 ± 0.07	4.79 ± 0.06	4.93 ± 0.02
	10	6.10 ± 0.05	4.96 ± 0.04	4.87 ± 0.03	5.12 ± 0.02	4.88 ± 0.02	4.86 ± 0.06	4.85 ± 0.03
	20	7.12 ± 0.03	5.11 ± 0.06	4.89 ± 0.02	5.29 ± 0.02	4.97 ± 0.02	4.95 ± 0.04	4.95 ± 0.03
	40	9.06 ± 0.06	5.73 ± 0.06	5.26 ± 0.04	5.84 ± 0.03	5.31 ± 0.01	5.24 ± 0.03	5.49 ± 0.02
Analysis	1	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07	4.73 ± 0.07
	2	4.90 ± 0.05	4.74 ± 0.01	4.74 ± 0.04	4.72 ± 0.04	4.79 ± 0.04	4.68 ± 0.05	4.85 ± 0.04
	4	5.30 ± 0.07	4.79 ± 0.06	4.73 ± 0.04	4.69 ± 0.06	4.69 ± 0.04	4.72 ± 0.03	4.81 ± 0.04
	10	6.10 ± 0.05	4.96 ± 0.04	4.80 ± 0.03	4.82 ± 0.05	4.83 ± 0.03	4.80 ± 0.05	4.83 ± 0.03
	20	7.12 ± 0.03	5.11 ± 0.06	4.91 ± 0.05	4.90 ± 0.02	4.90 ± 0.05	4.96 ± 0.02	4.96 ± 0.06
	40	9.06 ± 0.06	5.73 ± 0.06	5.27 ± 0.03	5.26 ± 0.02	5.34 ± 0.03	5.26 ± 0.04	5.57 ± 0.02

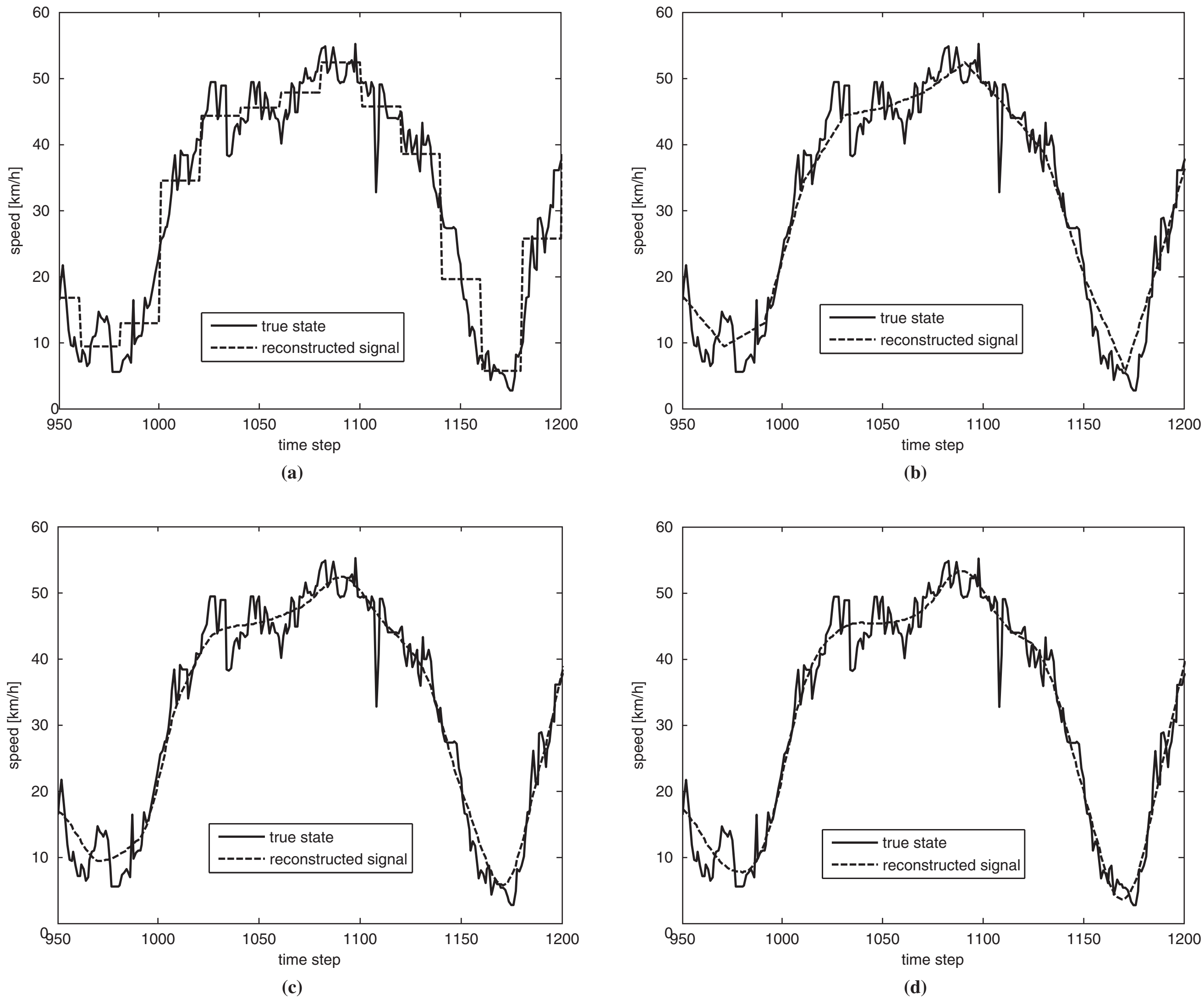

FIGURE 3 True state and reconstructed measurements for different interpolation approaches for cell with sensor: (*a*) stepwise approach, (*b*) linear interpolation approach, (*c*) cubic Hermite interpolation approach, and (*d*) optimization approach.

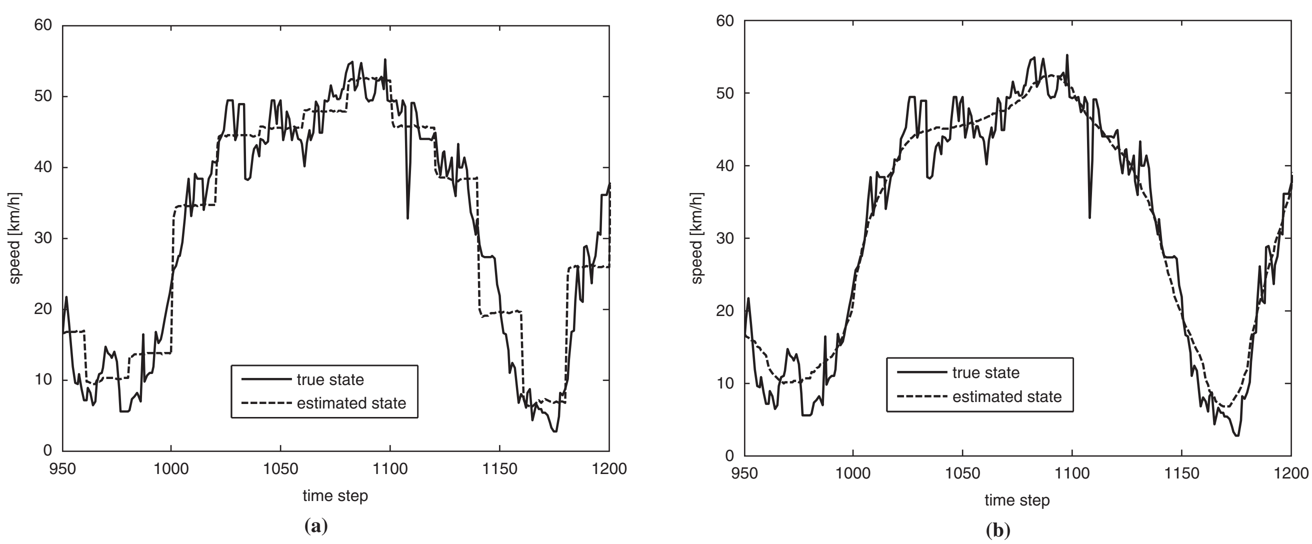

FIGURE 4 True and estimated speed for stepwise and cubic interpolation approaches for (*a*, *b*) cells with sensor.

(continued)

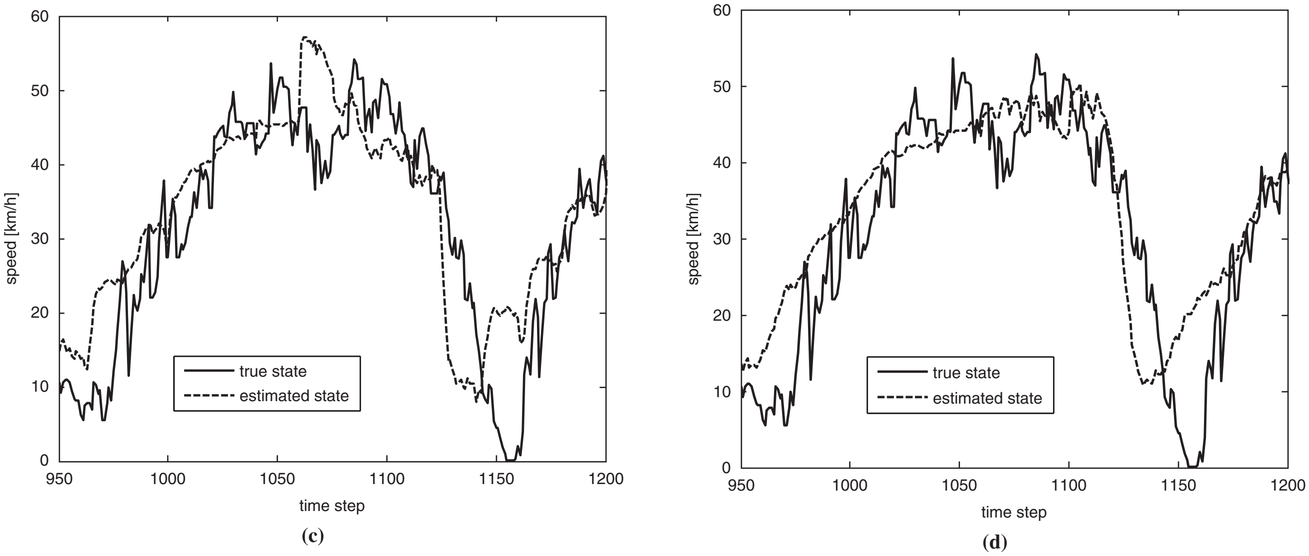

FIGURE 4 *(continued)* **True and estimated speed for stepwise and cubic interpolation approaches for (*c, d*) cells without sensor.**

to model short, sudden jumps in speed at time step 1.100 because of the aggregation of measurements. However, when the approaches are applied to the cell without a sensor, they are both less accurate since measurements are not available for this cell, and the correction step is never performed.

Table 2 reports the MAEs for different types of spatial cells. More specifically, Cell 1 represents the error for all cells that have virtual detectors, Cell 2 for all cells that are located immediately downstream of the cells with detectors, and Cell 5 is for cells located immediately upstream of the cells with sensors. From Table 2, it can be concluded that most approaches achieve the lowest error for cells with detectors and the highest error for the cells located the furthest from the detectors. Cells that are located downstream of sensors have lower error than cells located upstream of sensors. This finding can be explained by the fact that there are several backward shocks in the data set, which the CTM-v model reproduces well. The classic approach has a very high error for all cells, even cells in which the detectors are located. It is important to mention that the ghost cells modeling boundary conditions are not taken into consideration for this analysis.

TABLE 2 Reported MAEs for Types of Cells

Variable	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5
Classic	6.2644	6.1148	6.5151	6.0258	5.9105
Stepwise	2.8099	4.8487	5.8010	5.2290	4.2147
Linear interpolation	2.5477	4.5713	5.4521	4.8191	4.0003
Spline interpolation	3.3407	4.7495	5.5812	4.9976	4.2339
Hermite interpolation	2.7161	4.5584	5.4618	4.9266	4.0393
Optimization	2.4729	4.5551	5.4806	4.8766	4.0037
Kernel regression	2.6487	4.6113	5.3968	4.7949	4.0145

CONCLUSION

In the traffic domain, measurements at every time step are typically not available; instead, only aggregated measurements over predefined aggregation intervals are given. This procedure can pose a problem when traffic states are estimated, since the popular KF methods achieve state-of-the-art performance only when the measurements are available without gaps. To solve this issue, it was proposed to reconstruct the high-resolution measurement by using several approaches. It was further demonstrated how the reconstructed signal is used with the EnKF employing the CTM-v in two modes of operation: online one-interval delay mode and offline analysis mode. The results show the benefits of the proposed approach, which outperformed the existing methods and improved the accuracy of the underlying traffic model.

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