Analysis and Modeling of Social and Information Networks
CIS 4524/5524, Spring 2021

Assignment 4, due February 25 by 5pm on Canvas

Problem 1.

In the small-world network model, each of *n* nodes on a regular ring lattice is connected to *c* of its closest neighbors on the lattice (where *c* is even), and each of the edges of the lattice is rewired with probability p; by rewired, we mean that an edge on the ring is selected (with probability *p*), removed from the ring, and replaced with an edge that "crosses the ring" by joining two vertices chosen uniformly at random; such randomly placed edges are commonly referred to as *shortcuts*.

- What is the number of shortcut edges in a small-world network with *n* nodes and average degree of *c*?
- What is the number of shortcut *ends*?
- In the model described above, it is possible for a vertex to become disconnected from the rest of the network by the rewiring process, e.g., if all of the edges indecent to the vertex are rewired and no shortcut *ends* fall at the vertex. Show that the probability of this happening to a given vertex is $[pe^{-p}]^c$.

  (Hint: Recall that $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \frac{1}{e}$)

Problem 2.

Compute the expected maximum degree for the *Movie Actors* and for *Citation* network listed in this table:

Network	Size	$\langle k \rangle$	κ	γ_{out}	γ_{in}
WWW	325 729	4.51	900	2.45	2.1
WWW	4×10^7	7		2.38	2.1
WWW	2×10^8	7.5	4000	2.72	2.1
WWW, site	260 000				1.94
Internet, domain*	3015–4389	3.42–3.76	30–40	2.1–2.2	2.1–2.2
Internet, router*	3888	2.57	30	2.48	2.48
Internet, router*	150 000	2.66	60	2.4	2.4
Movie actors*	212 250	28.78	900	2.3	2.3
Co-authors, SPIRES*	56 627	173	1100	1.2	1.2
Co-authors, neuro.*	209 293	11.54	400	2.1	2.1
Co-authors, math.*	70 975	3.9	120	2.5	2.5
Sexual contacts*	2810			3.4	3.4
Metabolic, *E. coli*	778	7.4	110	2.2	2.2
Protein, *S. cerev.**	1870	2.39		2.4	2.4
Ythan estuary*	134	8.7	35	1.05	1.05
Silwood Park*	154	4.75	27	1.13	1.13
Citation	783 339	8.57			3
Phone call	53×10^6	3.16		2.1	2.1
Words, co-occurrence*	460 902	70.13		2.7	2.7
Words, synonyms*	22 311	13.48		2.8	2.8